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Still on the non-minimal versions of $N = 1$ supergravity

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Abstract. We show that a recently found non-minimal version of $N = 1$ Poincaré supergravity with (28+28) degrees of freedom is reducible to the non-minimal Breitenlohner set, in agreement with our earlier result that there exist at most five irreducible versions with auxiliary fields of spin up to one.

One way of understanding Poincaré supergravity is through a superconformal theory broken by the introduction of scale or compensating multiplets, the several off-shell versions being due to different choices of these multiplets (Kaku *et al* 1978, Kaku and Townsend 1978, Townsend and van Nieuwenhuizen 1979a, b). Recently, Kugo and Uehara (1982) have applied the superconformal theory to obtain the non-linear structure of the two new (20+20) non-minimal sets found by the authors (Rivelles and Taylor 1982a), and have also claimed that many more irreducible sets of auxiliary fields could be built by considering extra scale multiplets, with a (28+28) set given as an example (Kugo and Uehara 1982). This is in contradiction with our result that at most five irreducible sets of auxiliary fields with spins up to one can exist (Rivelles and Taylor 1982b, see also Sohnius and West (1983) for one of these sets). We elucidate the question by showing that the (28+28) set is in fact *reducible* to a particular form of the (20+20) non-minimal Breitenlohner set (Breitenlohner 1977a, b) and, contrary to Kugo and Uehara (1982) arguments, all other sets (apart from, at most, the five irreducible ones) should be reducible.

The scale multiplets which give rise to the known sets of auxiliary fields are: chiral (old minimal set (Stelle and West 1978, Ferrara and van Nieuwenhuizen 1978)), real linear (new minimal set (Sohnius and West 1981)), complex linear (Breitenlohner (1977a, b) non-minimal set), chiral and real vector (first new non-minimal set (Rivelles and Taylor 1982a)) and real linear and real vector (second new non-minimal set (Rivelles and Taylor 1982a)). The Poincaré supergravity action is then obtained by a choice of the scale multiplet and a set of gauge fixing conditions (Kaku *et al* 1978, Kaku and Townsend 1978, Townsend and van Nieuwenhuizen 1979a, b, Kugo and Uehara 1982). On the other hand the different sets of auxiliary fields found by Rivelles and Taylor (1982b; for one of these sets see Sohnius and West (1983)) were shown to be due to different choices of auxiliary irreps together with the condition that the supersymmetry transformation rules for the redefined fields be free of non-local terms. The aforementioned sets of auxiliary fields correspond, respectively, to the following choices of auxiliary irreps (0), $(\frac{1}{2}{}^{+}_{A})$, $(0, \frac{1}{2}{}^{+}_{A}, \frac{1}{2}{}^{-}_{A})$, $(0, 0, \frac{1}{2}{}^{+}_{A})$ and $(0, \frac{1}{2}{}^{+}_{A}, \frac{1}{2}{}^{+}_{A})$ ($\gamma^{\pm}_{A}$ means an irrep with superspin γ and parity ± 1 for the boson fields; for more details see Rivelles and Taylor (1983a) (see also Sohnius and West 1983)). The correspondence between the two formulations is easily seen since the irrep content of the scale multiplets coincides precisely with those of the auxiliary irreps in each case. In detail the irrep content of the scale multiplets is the following: chiral (0), real linear $(\frac{1}{2}{}^{+}_{A})$, complex linear $(0, \frac{1}{2}{}^{+}_{A}, \frac{1}{2}{}^{-}_{A})$ and real vector $(0, \frac{1}{2}{}^{+}_{A})$.

For the (28+28) set presented by Kugo and Uehara (1982) two scale multiplets are needed: the complex linear and the real vector. However, the last one is a submultiplet of the first one (as their irrep content shows), so that the real vector multiplet can be set to zero (without going on-shell) showing the reducibility of the model. Notice that this cannot be done with the new non-minimal sets since after setting either the chiral or real linear multiplets to zero we would end up with an even number of fermions.

The reducibility of the (28+28) model can also be shown at the component level. First we note the following misprints in Kugo and Uehara (1982): the last terms in equations (35, *d*, *e*) have the wrong sign; in equation (35, *c*) $Z \to Z_2$; in equation (35, *e*) the last Z must be replaced by Z_2 and in equation (35, *n*) the factor $\frac{7}{2}$ should be replaced by 2, as can be seen, for example, from requiring invariance of the Lagrangian of equation (33) of Kugo and Uehara (1982).

We can now set $Z = 0$ which implies $H = K = B_\mu = C = 0$. All these last equations are equivalent and allow λ_0 to be solved for in terms of the other fermions. This solution is uniquely given by replacing λ_0 by the spinor Λ in the real vector multiplet $\boldsymbol{V}$ and the scalar f by the corresponding scalar in that multiplet. Thus we take (in the notation of Kugo and Uehara (1982)) the combinations

$$\begin{aligned} \Lambda &= \lambda_0 - \tfrac{3}{2}(\beta/\alpha)\lambda_1 + \tfrac{3}{4}(\beta/\alpha)\mathrm{i}\gamma_5 \not\partial Z_2 + \tfrac{1}{2}(\beta/\alpha)\mathrm{i}\gamma_5 \boldsymbol{\gamma}\cdot\boldsymbol{R} - \tfrac{1}{2}\not\partial Z \\ D &= f - \tfrac{1}{2}(1 - \tfrac{3}{2}\beta^2/\alpha)\Box C - \tfrac{3}{2}(\beta/\alpha)\partial^\mu B^2_\mu + \tfrac{1}{2}(\beta/\alpha)(\Box\eta^{\mu\nu} - \partial^\mu\partial^\nu)h_{\mu\nu} \end{aligned} \tag{1}$$

and then we may set $\Lambda = D = 0$ in addition to the above. This is clearly erasing all of the (8+8) components $(C, Z, H, K, B_m, \Lambda, D)$ of $\boldsymbol{V}$. This erasure can be done without going on-shell, since the fields in $\boldsymbol{V}$ only transform into each other, and setting them to zero does not disturb the transformation laws of the remaining fields. Furthermore, the transformations of the set $\boldsymbol{V}$ preserve the condition $\boldsymbol{V} = 0$. The replacement (1) in the linearised Lagrangian is also straightforward. The new terms in the Lagrangian involving the non-zero fields λ_1, Z_2, ψ_μ, $h_{\mu\nu}$ in (1) always appear when multiplied either by C or Z; they therefore disappear on setting $\boldsymbol{V}$ to zero. Thus the subspace is an invariant subspace, and the (28+28) representation is reducible.

The transformation rules now reduce (using the notation of Kugo and Uehara (1982) with $\gamma_5^2 = +1$, $\sigma_{\mu\nu} = \frac{1}{4}[\gamma_\mu, \gamma_\nu]_-$, and metric diag (1, 1, 1, 1)) to

$$\begin{aligned} \delta h_{\mu\nu} &= \tfrac{1}{2}\bar{\varepsilon}\gamma_{(\mu}\psi_{\nu)} \\ \delta\psi_\mu &= D_\mu\varepsilon - \tfrac{3}{4}\mathrm{i}\gamma_5\varepsilon A_\mu - \tfrac{1}{2}\gamma_\mu(H_1 + \mathrm{i}\gamma_5 H_2 + \mathrm{i}\gamma_5 \tilde{\not B} + \tfrac{1}{2}\mathrm{i}\gamma_5 \not A)\varepsilon \\ \delta H_1 &= \tfrac{1}{2}\bar{\varepsilon}\mathrm{i}\gamma_5\lambda_1 + \tfrac{1}{4}\bar{\varepsilon}\not\partial Z_2 + \tfrac{1}{12}\bar{\varepsilon}\boldsymbol{\gamma}\cdot\boldsymbol{R} \\ \delta H_2 &= -\tfrac{1}{2}\bar{\varepsilon}\lambda_1 + \tfrac{1}{4}\bar{\varepsilon}\mathrm{i}\gamma_5\not\partial Z_2 + \tfrac{1}{12}\bar{\varepsilon}\mathrm{i}\gamma_5\boldsymbol{\gamma}\cdot\boldsymbol{R} \\ \delta\tilde{B}_\mu &= -\tfrac{1}{2}\bar{\varepsilon}\gamma_\mu\lambda_1 - \tfrac{1}{2}\bar{\varepsilon}\mathrm{i}\gamma_5\sigma_{\mu\nu}\partial^\nu Z_2 - \tfrac{3}{4}\bar{\varepsilon}\mathrm{i}\gamma_5\partial_\mu Z_2 - \tfrac{1}{6}\bar{\varepsilon}\mathrm{i}\gamma_5\gamma_\mu\boldsymbol{\gamma}\cdot\boldsymbol{R} \\ \delta A_\mu &= \bar{\varepsilon}\mathrm{i}\gamma_5\partial_\mu Z_2 - \tfrac{1}{2}\bar{\varepsilon}\mathrm{i}\gamma_5 R_\mu + \tfrac{1}{6}\bar{\varepsilon}\mathrm{i}\gamma_5\gamma_\mu\boldsymbol{\gamma}\cdot\boldsymbol{R} \\ \delta B^2_\mu &= \tfrac{1}{2}\bar{\varepsilon}\mathrm{i}\gamma_5\gamma_\mu\lambda_1 - \tfrac{1}{4}\bar{\varepsilon}\not\partial\gamma_\mu Z_2 \\ \delta Z_2 &= (H_1 + \mathrm{i}\gamma_5 H_2)\varepsilon + \tfrac{1}{2}(\mathrm{i}\gamma_5\tilde{\not B} - \not B_2)\varepsilon \\ \delta\lambda_1 &= -\tfrac{1}{2}\sigma^{\mu\nu}\partial_\mu\tilde{B}_\nu\varepsilon - \tfrac{1}{4}\partial^\mu(3\tilde{B}_\mu + 2A_\mu)\varepsilon + \tfrac{1}{4}\mathrm{i}\gamma_5\gamma^\mu\not\partial B^2_\mu\varepsilon + \tfrac{1}{2}\not\partial(\mathrm{i}\gamma_5 H_1 - H_2)\varepsilon \end{aligned} \tag{2}$$

and the action is now

$$I_{SG}=\int d^4x[-\tfrac{1}{2}(R+\bar{\Psi}\cdot R)+3(H_1^2+H_2^2)-\tfrac{3}{2}\tilde{B}_\mu^2+\tfrac{3}{4}A_\mu^2-\tfrac{3}{2}(B_\mu^2)^2-3\bar{Z}_2 i\gamma_5\lambda_1].$$

To show the closure of the algebra we just need to compare this model with the Breitenlohner set of auxiliary fields (Breitenlohner 1977a, b). Since, as far as we know, the transformation rule for this last set has not been published (Rivelles 1982) in its full form (with the n dependence) we will give it here. The transformation rules and action are (Rivelles 1982)

$$\begin{aligned}
\delta h_{\mu\nu} &= \tfrac{1}{2}\bar{\varepsilon}\gamma_{(\mu}\psi_{\nu)}\\
\delta\psi_\mu &= D_\mu\varepsilon+\tfrac{1}{4}n^{-1}i\gamma_5\varepsilon[(3-n)g_\mu-3(1-n)a_\mu]-\tfrac{1}{2}i\sigma_{\mu\nu}\gamma_5\varepsilon g^\nu+\tfrac{1}{6}\gamma_\mu(\varepsilon f-i\gamma_5\varepsilon g)\\
\delta g_\mu &= \tfrac{1}{6}(3-n)\bar{\varepsilon}i\gamma_5 R_\mu-\tfrac{1}{3}n\bar{\varepsilon}i\sigma_{\mu\nu}\gamma_5 R^\nu+\tfrac{1}{2}\bar{\varepsilon}i\sigma_{\mu\nu}\gamma_5\partial^\nu\lambda+\tfrac{1}{4}\bar{\varepsilon}i\gamma_5\partial_\mu\lambda+\tfrac{1}{4}\bar{\varepsilon}i\gamma_\mu\gamma_5\chi\\
\delta a_\mu &= \tfrac{1}{2}\bar{\varepsilon}i\gamma_5 R_\mu+\tfrac{1}{4}(1-n)^{-1}(2\bar{\varepsilon}i\sigma_{\mu\nu}\gamma_5\partial^\nu\lambda-\bar{\varepsilon}i\gamma_5\partial_\mu\lambda+\bar{\varepsilon}i\gamma_\mu\gamma_5\chi)\\
\delta v_\mu &= \tfrac{1}{2}\bar{\varepsilon}\sigma_{\mu\nu}\partial^\nu\lambda-\tfrac{1}{4}\bar{\varepsilon}\partial_\mu\lambda+\tfrac{1}{4}\bar{\varepsilon}\gamma_\mu\chi\\
\delta f &= \tfrac{1}{4}n\bar{\varepsilon}\gamma\cdot R-\tfrac{3}{8}(\bar{\varepsilon}\not{\partial}\lambda-\bar{\varepsilon}\chi)\\
\delta g &= -\tfrac{1}{4}n\bar{\varepsilon}i\gamma_5\gamma\cdot R-\tfrac{3}{8}(\bar{\varepsilon}i\not{\partial}\gamma_5\lambda+\bar{\varepsilon}i\gamma_5\chi)\\
\delta\lambda &= \tfrac{1}{2}(1-n)i\gamma^\mu\gamma_5\varepsilon(g_\mu-a_\mu)-\tfrac{1}{2}\gamma^\mu\varepsilon v_\mu-\tfrac{1}{3}(1-n)(\varepsilon f-i\gamma_5\varepsilon g)\\
\delta\chi &= -(1-n)i\sigma^{\mu\nu}\gamma_5\varepsilon\partial_\mu(g_\nu-a_\nu)-\tfrac{1}{2}(1-n)i\gamma_5\varepsilon\partial^\mu(g_\mu+a_\mu)\\
&\quad -\sigma^{\mu\nu}\varepsilon\partial_\mu v_\nu+\tfrac{1}{2}\varepsilon\partial\cdot v+\tfrac{1}{3}(1-n)\not{\partial}(\varepsilon f-i\gamma_5\varepsilon g)
\end{aligned}\tag{3}$$

$$I_{SG}=\int d^4x\{-\tfrac{1}{2}(R+\bar{\Psi}\cdot R)+\tfrac{3}{4}n^{-1}[g_\mu^2-(1-n)a_\mu^2+(1-n)^{-1}v_\mu^2-\tfrac{4}{9}(f^2+g^2)-(1-n)^{-1}\bar{\lambda}\chi]\}$$

with $-\infty<n<\infty$, $n\neq 0, 1$. After performing the following field redefinitions in (2): $H_1=-\frac{1}{3}f$, $H_2=\frac{1}{3}g$, $A_\mu=g_\mu-2a_\mu$, $\tilde{B}_\mu=a_\mu-g_\mu$, $B_\mu^2=\frac{1}{2}v_\mu$, $\lambda_1=\frac{1}{4}i\gamma_5\chi$, and $Z_2=\frac{1}{2}\lambda$, we obtain the set (3) with $n=-1$, where our parameter n is related to that of Siegel and Gates (1979), n^{SG}, by $n^{SG}=-\frac{1}{3}n$. Thus the set (2) is automatically closed and it is a version of the Breitenlohner set.

We can apply the same argument to show the reducibility of the other proposed version (Kugo and Uehara 1982) involving one chiral and three real vector scale multiplets: it should reduce to the first new non-minimal set (Rivelles and Taylor 1982a). Thus, just adding on new scale multiplets and choosing gauge fixing conditions does not necessarily produce new irreducible sets of auxiliary fields. The number of such irreducible sets remains at most five.

We add finally that if we modify our definition of reducibility so as to allow for multiples of irreducible multiplets to be regarded as reducible to the irreducible multiplet itself, then even the two new cases (Rivelles and Taylor 1982a) are reducible; we discuss this in detail elsewhere (Rivelles and Taylor 1983b).

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